Dealing with Lashkar-e-Taiba: A Multi-Player Game-Theoretic Perspective

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Abstract-Lashkar-e-Taiba (LeT) is one of the deadliest terrorist groups in the world. With over 100 attacks worldwide since 2004, LeT has become a political force within Pakistan, a proxy fighting force for the Pakistani Army, and a terror group that can carry out complex, coordinated attacks such as the 2008 Mumbai attacks. In this paper, we develop a game-theoretic analysis of how to deal with LeT using a 5-player game whose players include LeT, India, the Pakistani military, the (civilian) Pakistani government, and the US. We use an expert on LeT and Pakistan to develop a payoff matrix and compute pure and mixed Nash equilibria (NE) in this payoff matrix. We study several of these NEs in detail. Our analysis shows that: (i) there are 6 pure NEs in which LeT eliminates its armed wing, (ii) increasing external financial/military support for Pakistan leads to no NEs where LeT reduces violence, (iii) almost all NEs in which LeT significantly reduces violence involve coordinated actions by both the US and India.

I. INTRODUCTION

Lashkar-e-Taiba (LeT) is one of the deadliest terrorist groups in the world. With over 100 attacks worldwide since 2004 that have taken over 700 lives (according to the US National Counterterrorism Center's Worldwide Incident Tracking System¹), LeT is a political force within Pakistan, a proxy fighting force for the Pakistani Army, and a terror group that can carry out complex, coordinated operations such as the 2008 Mumbai attacks. After Osama bin Laden's death in May 2011, LeT's leaders led prayer marches in sympathy to bin Laden's jihad.

In this paper, we develop a multi-player game-theoretic approach to the problem of reining in **LeT**'s harmful activities based on non-zero-sum games.² Our framework studies five "players" who are all centrally involved in Pakistan:

- LeT itself;
- The US (which in our model is really a proxy for both the United States and allied Western powers),
- India,
- The Pakistani *military*, which is a proxy for not only Pakistan's military, but also its intelligence agencies, national security and paramilitary forces,

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• The Pakistani *civilian* government which is construed broadly to include the current government and its supporters, opposition parties in Parliament, and key civilian institutions in the country.

We are not in a position to model every entity wielding influence in Pakistan and/or on **LeT**. This is why the "players" listed above are proxies for a set of players. For each of these players, we study a small set of actions. In the case of the US, we study the actions of: DO NOTHING, COVERT ACTION against **LeT** (*e.g.*, special operations, sabotage against **LeT** facilities), CUT SUPPORT (economic, military) to Pakistan, and EXPAND SUPPORT (economic, military) to Pakistan. Thus there is a mix of carrots and sticks in the set of US options towards Pakistan. A suite of similar actions involving both carrots and sticks is studied for the other four players as well.

An expert on the geopolitics of south Asia and **LeT** in particular then created a *payoff matrix* specifying the payoff for each of the 5 players in each of the 768 possible combinations of actions. The payoff for each player was specified for each possible combination of actions taken by the 5 players in the study and was given on a -2 to +2 scale (-2 being least preferred, +2 being most preferred). This expert had no knowledge of game theory and no idea what would be computed with this payoff matrix. This accounts for personal bias; although future work will include multiple experts.

We computed the entire set of pure Nash equilibria (NEs) for this payoff matrix. There were 24 pure NEs—in addition, we found 1892 mixed NEs, making a total of 1916 NEs that we considered. Due to space restrictions, we are focusing on the 13 NEs in which LeT eliminates its armed wing (most preferred) with high probability and launches no major violent operations. Our findings are summarized below.

Finding 1. The US and India should carefully coordinate actions against LeT. In 12 of these 13 NEs, the US takes covert action against LeT (the one remaining case requires they cut support to Pakistan). Likewise, all 13 NEs require that India either take covert action against LeT or coercive diplomatic action against Pakistan. Jointly coordinated (at the strategic, not tactical level) operations seem to have the desired effect on LeT, according to our game-theoretic models.

Finding 2. US financial and military support to Pakistan has not helped curb LeT violence. There are no pure NEs in which the US expands aid to Pakistan. While there are a total of 8 equilibria altogether (of the total 1916 considered) in

http://www.nctc.gov/site/other/wits.html

²This follows seminal studies by Schelling [1, p.269] in which he says "Actually, it is hard to see how military relations can ever even approximate a zero-sum game unless things have reached the stage where all but two extremes among the possible outcomes have become—for reasons of diplomacy, personality, technology, and geography, or some profound incompatibility between the two sides—practically unattainable and irrelevant to decision.

which the US expands aid to Pakistan, all 8 involve undesired actions from **LeT** and Pakistan. In all 8 of these situations, the Pakistani military expands support for **LeT**, the Pakistani government either endorses **LeT** or does nothing, and **LeT** launches major violent operations.

Finding 3. LeT cannot be defeated without support from the Pakistani military. Of course, this finding has been long suspected. Our mathematical and/or game-theoretic perspective confirms this. 10 out of the 13 NEs require Pakistan's military to either crack down on LeT or cease supporting them in order to deterministically achieve no violent attacks. However, if we only want to achieve at least a 0.5 or 0.333 probability, then 11 or, respectively, all 13 NEs say that the Pakistani military must somehow be persuaded to at least cease supporting LeT. Fortunately, findings (1) and (2) above help.

Finding 4. Pakistan's civilian government can remain passive. Our game-theoretic model shows that the Pakistani civilian authority must either do nothing or they must prosecute LeT's leaders. As it is vastly easier to do nothing, we suspect this would be the route taken.

The paper is organized as follows. Section II describes the players and actions in our model and explains how the payoff matrix was created. Section III reviews Nash equilibria and explains the 13 **NE**s we discovered in which **LeT** does not launch any major violent operations and eliminates its armed wing with high probability. Section IV reviews related work and concludes with recommendations for US policy makers.

II. OUR MULTI-PLAYER MODEL

In this section, we describe our multi-player model. We discuss the actions we considered for each player. *In addition to the actions explicitly discussed, each player also has an action called* NONE *which corresponds to doing nothing.*

A. US Actions

1) Covert action against LeT: Covert action against LeT could be implemented in many ways—we are not suggesting any specific covert action. Such actions could include sabotaging the water and/or electricity supply to LeT training camps and specific targeted action (*e.g.*, capture or assassination) of key LeT operatives. The ongoing drone strikes against Pakistani Taliban leaders, which the Pakistani government officially criticizes but privately supports, and the successful operation to kill al-Qaeda leader Osama bin Laden indicate that the United States possesses the capability to carry out covert operations in Pakistan. Furthermore, the Raymond Davis imbroglio in which a CIA contractor who was carrying detailed information about LeT was arrested by Pakistani police after killing a pair of Pakistanis suggests that the United States is at least considering covert options against LeT.

2) Cut support for Pakistan: According to the Congressional Research Service, the US appropriated \$2.735 billion (including Coalition Support Funds) in military aid and \$1.727 billion in economic aid to Pakistan in FY2010.³ Since the operation that killed Osama bin Laden, there is much debate

³See "Pakistan-U.S. Relations: A Summary," prepared by K. Alan Kronstadt of the Congressional Research Service, May 16, 2011.

within the US government about whether such aid should continue.⁴ Support for Pakistan can be cut in other ways (*e.g.*, by increasing tariffs on textiles, Pakistan's major export). We have thus included this as one possible action the US can take.

3) Expand support for Pakistan: There have also been suggestions in the US government that Pakistan should receive increased amounts of development aid so that the money in question can be used to build civilian institutions and help improve the quality of education (and in the process wean students away from institutions of learning that promote radical agendas).⁵ Expanded support for Pakistan could also include support for Pakistani textile exports to the US, support for Pakistani requests for various types of advanced technology, and so forth. We include this as an option as well.

In addition, NONE is an action denoting a conscious decision by the US not to do anything new. It is important to note that we are not recommending one of these actions or another.

B. India Actions

As in the case of the US, we study three actions that India might take—and as in the above case, there are many ways in which India could tactically implement these actions.

1) Covert action against LeT: This action is similar to the corresponding action listed above for the US.

2) Coercive diplomacy: The current Indian government has shown considerable restraint in dealing with calls for retaliation after the Mumbai attacks. Previous governments likewise exercised restraint after various attacks that are believed to be **LeT**-backed. India could use *coercive diplomacy*, credible threats, to affect Pakistan and **LeT** adversely. For instance, India could choose to violate (or threaten to violate) the Indus Water Treaty the next time a terror attack is linked to **LeT** which India believes is supported by the Pakistani military—an opinion shared by many other third parties [2]. Such an action would adversely affect Pakistan's water supply. Other examples of coercive diplomacy include India's military deployment after the December 2001 attacks on the Indian Parliament (see [3] for discussion) or a major international campaign to diplomatically isolate Pakistan.

3) Propose shared sovereignty in Kashmir: Kashmir remains the biggest bone of contention between India and Pakistan. Over the years, there have also been many calls for shared sovereignty in the disputed territories of Jammu and Kashmir (J & K) [4]. Public calls for this arrangement have mostly come from the Pakistani side and have usually been firmly rejected by India. However, it has been suggested that India consider some form of shared governance in Kashmir.

C. Pakistan Military Actions

We study three possible actions for the Pakistani military to take, all related to their support for **LeT**.

⁴On May 3, 2011, H.R. 1699, the Pakistan Foreign Aid Accountability Act, was introduced in the House of Representatives. The Act would prohibit future foreign assistance to Pakistan unless the Secretary of State certifies that the Pakistani government was not complicit in hiding Osama bin Laden.

⁵See "Afghanistan and Pakistan Regional Stabilization Strategy," prepared by the Office of the Special Representative for Afghanistan and Pakistan 2010. 1) Crack down on LeT: A crackdown on LeT would mean the arrest of members of LeT and/or an effort to close down LeT's training camps, shutting down the logistics support for LeT operations in J & K, and taking steps to interdict LeTallied organizations like Jamaat-ud-Dawa. Pakistani security has at times cracked down on LeT, particularly when it becomes the target of international notoriety. After the December 2001 parliament attack and the November 2008 attacks on Mumbai, Pakistani authorities arrested hundreds of LeT operatives and closed dozens of LeT offices.

2) Cease support for LeT: This is a weaker action in which the Pakistani military does not crack down on LeT, but takes overt and visible steps to stop any support for LeT. This could include arrests of military officers found to be supporting LeT illicitly, stopping any training programs for LeT conducted by military officers, and so forth.⁶

3) Expand support for LeT: A third option for the Pakistani military is to expand support for LeT by providing additional training to LeT operatives, by expanding logistical and materiel support for LeT operations (as David Headley has recently alleged in a Chicago courtroom was the case with the Mumbai attacks). See [5] for further discussion.

D. Pakistan Government Actions

We consider just two possible actions by the civilian side of the Pakistani government (excluding the military side).

1) Prosecute LeT's Leaders: India and the US have frequently called for LeT's leader Hafeez Saeed and other key leaders like Zaki-ur-Rahman Lakhvi to be arrested, prosecuted and possibly extradited. Pakistan has steadfastly refused to do this, though they have occasionally put LeT leaders under house arrest, and arrested (but usually subsequently released) some LeT members. One option for the Pakistani civilian government is to aggressively prosecute key LeT leaders.

2) Endorse LeT's Social Services: LeT provides a range of social services including running schools, medical clinics, and disaster relief programs. Like Hamas in the West Bank and Hezbollah in Lebanon (studied in [6], [7]), LeT's social service programs earn the respect of the community among large swathes of the Pakistani population, facilitating the recruitment of terrorists. The Pakistani civilian government, which frequently clashes with jihadist and fundamentalist groups, could ratchet up support for LeT's social services (which do fill a void), gaining an ally for themselves and increasing support for their government within the local populace. It has been reported that a US diplomatic cable highlights the domestic politics surrounding taking legal measures against LeT. In this cable, the US ambassador describes a situation in which the Pakistani government has difficulties prosecuting LeT's leader Hafiz Saeed because the main opposition party hampers the government's efforts in order to gain popular support.

We are not suggesting Pakistan's government choose one of these options—these are merely actions that we believed they would consider, and hence, we consider them in the study.

E. Lashkar-e-Taiba's Actions

In the case of **LeT**, we considered three actions (in addition to the NONE action considered for all players in the model).

1) Launch major violent attacks: LeT has shown the ability to launch major violent attacks on Indian soil. Spectacular attacks launched by LeT in the past include the November 2008 Mumbai siege in which five sites were targeted in a coordinated attack, the Mumbai train bombings of July 2006 in which over 200 people were killed in a series of seven blasts, and the September 2002 attack on Akshardham Temple in which over 20 were killed. They are also frequently cited as participants in an attack on the Indian Parliament in 2001, though this is more commonly attributed to the Pakistan-based terror group, Jaish-e-Mohammed. It is clear that LeT has the ability to launch major new violent attacks.

2) Hold violent operations: In addition to major attacks such as those mentioned above, LeT could carry out attacks at a lower level. These include the kinds they have typically carried out in J & K, which include operations against Indian government and security targets. In December 2001, after LeT was banned by the US, LeT stated that it was confining its efforts to J & K [8].

3) Eliminate armed wing: A dramatic action for LeT to take could include the renunciation of violence and the elimination of their armed wing. This would not happen quickly, but may happen if LeT political goals are met. Pakistani security officials have privately suggested that if the Kashmir issue were settled, LeT could be directed to abandon its armed wing and focus on providing social services [9]. Some American officials also believe this is a path to be explored.

III. GAME-THEORETIC FINDINGS

In order to run a game-theoretic analysis of the interactions between the five players described in the preceding section, we first had to create a payoff matrix [10]. An action situation is an assignment of actions to each player. In our LeT analysis, each of the players has either three or four actions (including the NONE action). Thus, there are a total of 768 possible action situations to analyze. A payoff matrix is a simple matrix whose rows correspond to action situations and whose columns correspond to players. Thus, the entry in the payoff matrix corresponding to action situation as_i and player p_i describes the *payoff* that player p_i gets if all the players choose the actions specified in action situation as_i . In our application, payoffs were expressed on a qualitative -2 to +2 scale with -2 denoting a terrible payoff (least preferred situation for player p_i) and a +2 denoting the best possible payoff. The payoff matrix in this application was created by an LeT and Pakistan expert who had no knowledge of game theory. However, he was asked to provide ratings on this scale, and he was told what the payoff values meant.

A *pure Nash equilibrium* (**NE** for short) intuitively reflects an action situation in which all players have made the best decision they can after taking into account the needs and payoffs of the other players. Simply put, a pure **NE** provides a single action for each agent, such that this action is the

⁶During a period of warming Pakistani-Indian relations in early 2004, the Pakistani military began demobilizing operatives of LeT and other proxy groups it has supported in Kashmir. However, it did not completely shut down their operations, only restricted them. See Mohammad Amir Rana, *The Seeds* of *Terrorism* (London: New Millennium, 2005), 283.

"best" assuming all other agents follow their pure **NE** actions. However, Nash equilibria also allow players to play what are called "mixed strategies." Informally speaking, what this means is that rather than choosing one action to perform in an action situation, a player can choose to perform one of several actions, each to be performed with some probability. Such (impure) Nash equilibria account for the fact that a player might want to insert some uncertainty into his actions so he can keep the other players "guessing." This is the intuition behind Nash equilibria; a formal technical definition can be found in almost any classical game theory book [10]. The use of payoff matrices was extensively studied by Tom Schelling in national security contexts [11], as well as in a variety of political science contexts [12].

A. Calculating Pure and Mixed Nash Equilibria

Game theory has traditionally focused on small games, typically limited to two players and a small set of actions. This focus is driven both by the inherent difficulties in proving theoretical results relating to complex games and, importantly, by the computational power required to find results empirically. While the methods for computing pure and mixed Nash equilibria are beyond the scope of this paper, advances in the last decade, coupled with increased computational power, have taken cursory steps toward solving larger games.

For our experiments, we exhaustively enumerated pure NEs and used a simplical subdivision solver to (inexhaustibly) compute 1892 mixed NEs [13]. We used the Gambit Software Toolkit [14] to enumerate completely or partially the Nash equilibria for this game. Some results are discussed below.

- LeT did not launch any major new violent attacks and the probability of LeT eliminating its armed wing was non-zero. We found 13 such NEs of which 6 were pure.
- 2) LeT did not launch any major new violent attacks with a probability under 0.25 and the probability of LeT eliminating its armed wing was non-zero. In addition to the 13 NEs in the previous case, we found another 5 NEs.
- 3) The US expanded aid to Pakistan—not a single pure NE included this option. When we allowed the US to expand aid to Pakistan with probability at least 0.5 as part of a mixed strategy, we found a total of 8 (impure) NEs whose details are summarized below.

B. Pure Equilibria Associated with LeT's Good Behavior

We now study the six pure equilibria associated with good behavior from LeT, *i.e.*, when LeT does not launch major violent armed attacks and where the probability that they eliminate their armed wing is high. The six equilibria E_1, \ldots, E_6 are given in Figure 1.

Eq.	US	India	Pak-Military	Pak-Government	LeT
E_1	Cut Support	Covert Ops	Cease Support	None	Eliminate A.W.
E_2	Covert Ops	Coercive Diplomacy	Cease Support	None	Eliminate A.W.
E_3	Covert Ops	Coercive Diplomacy	Cease Support	Prosecute LeT	Eliminate A.W.
E_4	Covert Ops	Coercive Diplomacy	Crackdown LeT	None	Eliminate A.W.
E_5	Covert Ops	Covert Ops	Cease Support	None	Eliminate A.W.
E_6	Covert Ops	Covert Ops	None	Prosecute LeT	Eliminate A.W.

Fig. 1. The six pure NEs containing good behavior from LeT.

We now discuss each equilibrium in greater detail below. **Eq.** E_1 . In this equilibrium, the US and India apply joint pressure—the US cuts support to Pakistan (*e.g.*, development aid or military aid or other kinds of support) and India concurrently starts low-level covert operations against LeT. These two actions are likely to put pressure on Pakistan financially at the same time that LeT's ability to operate is diminished through low-level covert operations against them. However, to bring this about, the Pakistani military will need to cease providing support to LeT, causing LeT to abandon its military wing due to the combined Indian covert operations on it and the lack of support from the Pakistani military. The main question involved in this NE is the issue of how to get Pakistan's military to stop providing support to LeT. There are several options-one is to provide financial rewarding overseas retirement packages to many leading Pakistani military officers supportive of LeT. Though in some ways rewarding bad behavior, this may be the cheapest and most effective way out and should be considered, at least, as an option.

Eq. E_2 . As in E_1 , the U.S. and India apply joint pressure but in this case the United States engages in covert operations against LeT while India pursues "coercive diplomacy," or threats of adverse consequences instead of negotiations. India has a number of options, including using its control over Pakistan's primary water sources, leading a major international campaign against Pakistan in the UN and other international bodies, and large-scale military maneuvers to which the Pakistani military would have to reply in kind, incurring costs that Pakistan can ill afford in its current economic situation. These strategies have potentially high costs to India if they fail to be successful or if they induce unpredictable behavior on the part of Pakistan. However, covert operations in their territory are deeply embarrassing to Pakistan's security establishment. Combined with an assertive challenge from India, Pakistan's military might find the cost of supporting LeT to be simply too high and start a crackdown. LeT, losing support from its patron and under pressure from American covert operations, might find that it simply cannot maintain its military operations and continue its social welfare mission.

Eq. E_3 . This equilibrium is similar to E_2 except that it adds the additional factor that Pakistan's civilian government is prosecuting LeT. Generally legal efforts against LeT on the part of Pakistan's government have been lackluster and stymied by the group's popularity within Pakistan. Faced with credible public action on the part of India as well as covert actions hampering LeT, the civilian government may be inclined to pursue more vigorous efforts. The threat of serious imprisonment (as opposed to the past informal house arrests) combined with other factors might persuade LeT's leadership to focus on social welfare activities and dispense with its armed wing.

Eq. E_4 . Under this situation, rather than ceasing support the Pakistani security establishment launches an extensive and sustained crackdown on LeT as India engages in coercive diplomacy and the United States pursues covert operations. As US covert operations and an inability to respond to Indian pressure are deeply embarrassing to Pakistan's national security establishment, the crackdown on LeT might effectively be punishment for bringing this humiliation down on Pakistan's generals. Again, the combination of American and Pakistani pressure could lead **LeT** to find that maintaining an armed wing comes at the cost of the organization as a whole.

Eq. E_5 . This equilibrium is like E_4 , only India joins the US in covert operations and the Pakistani military ceases support for **LeT**. The combination of actions under E_5 raise the cost of armed operations on **LeT**. Covert actions that hamper **LeT**'s ability to function and undermine its reputation, when combined with the loss of support from the Pakistani military, could effectively force **LeT** to choose between its armed operations and its very existence.

Eq. E_6 . In this equilibrium, as in the others, India and the US cooperate in carrying out covert operations against LeT. It is different in that the Pakistani military does nothing, but Pakistan's civilian government aggressively prosecutes LeT. This would be significant because in the past, when LeT attracted too much international attention, Pakistan's intelligence services have sought to preserve their favored proxy by advising LeT how to lower its profile by engineering pro forma splits between the military and charitable wings.

C. Mixed Equilibria Associated with LeT's Good Behavior

In addition to the six pure NEs in which LeT both renounced their armed wing and stopped launching attacks, there were seven mixed NEs that led to "almost" good behavior in the sense that they gave up launching major violent attacks, but where the probability of eliminating their armed wing was not 1. We study these equilibria below. We first note that *all seven* of these NEs shared some common aspects:

- 1) The US carried out covert actions (*i.e.*, adopted a pure strategy with no probabilities involved).
- India adopted a pure strategy as well—but in some of the seven equilibria, India opted to use covert action against LeT, while in others, it used coercive diplomacy.
- 3) Pakistan's military does not expand support for LeT.
- Pakistan's civilian government does not endorse LeT's social services program in any of these NEs.

These common threads allow us to infer that the joint application of pressure by India and the US on Pakistan (through covert actions and/or coercive diplomacy) is key to incentivizing responsible behavior not only from **LeT**, but also the Pakistani military. Figure 2 describes these equilibria.

		Mixed Nash Equilibrium						
Agent	Action	7	8	9	10	11	12	13
US Covert Ops		1	1	1	1	1	1	1
India	Covert Ops	0	0	0	0	0	1	1
India	Coercive Dip	1	1	1	1	1	0	0
	Crackdown	1	1	1	1/3	1/3	0	1/4
Pak-Mil	Cease Sup.	0	0	0	1/3	2/3	1	0
	None	0	0	0	1/3	0	0	3/4
Pak-Gov	Prosecute	1	1	1/2	1	19/24	1	1
	None	0	0	1/2	0	5/24	0	0
	Hold Ops.	1/2	1/3	0	0	0	1/2	1/2
LeT	Elim. A.W.	1/2	2/3	1/2	1/3	1/3	1/2	1/2
	None	0	0	1/2	2/3	2/3	0	0

Fig. 2. Probabilities per action for each of the seven mixed NEs with "good" behavior from LeT.

D. Summary

We start by first noting the following common aspects of our analysis with respect to the **NE**s that had **LeT** giving up their armed wing with non-zero probability.

- India and the US must work together to pressure the Pakistani government and LeT. All 13 NEs require both that the US cut support or carry out covert operations to undermine LeT and that India either carry out covert activities or practice coercive diplomacy towards Pakistan.
- 2) Not one of the 13 NEs said US financial support for Pakistan should be expanded.
- Not one of the 13 NEs said Pakistan's government should expand support for the social services provided by LeT.

In view of the first item above, it is clear that whether **LeT**'s harmful activities can be shut down or severely restricted depends very much on both the Pakistani military and the Pakistani civilian government. We therefore subjected the **NE**s to one further analysis.

We had multiple experts assign costs for each action, reflecting the difficulty of implementing a particular action. These costs are independent from the utilities used to create the payoff matrix for equilibria computation; rather, they represent the difficulty of performing an action, which is not built into the payoff utilities. For instance, one expert (for analysis A_1) set the cost of a crackdown on **LeT** to be 10, the cost of ceasing support for **LeT** to be 5, and the cost of prosecuting **LeT** leaders to be 10. All other costs of actions associated with non-**LeT** agents were set to 0. All three expert analyses are shown in Figure 3.

Analysis	US	India	India	Pak-Mil	Pak-Mil	Pak-Gov	
	Covert	Covert	Coer-	Crack-	Cease	Prose-	
	Ops	Ops	cion	down	Support	cute	
A_1	-	-	-	10	5	10	
A_2	-	-	-	10	4	5	
A_3	3	3	3	10	4	5	

Fig. 3. Expert-assigned costs applied to non-LeT actions.

The *expected cost* of an equilibrium is now computed like an expected value—the expected cost of an action is the probability of the action (in the equilibrium) times the cost of the action. The expected cost of an equilibrium is the sum of all of these costs.

At the same time, we also defined the *benefit* (to international security) of **LeT**'s actions. For example, the expert corresponding to analysis A_1 determined eliminating **LeT**'s armed wing has a benefit of 10, while holding small operations has a benefit of 5. All other actions have benefit 0. The *expected benefit* of an equilibrium is then computed in the same way as above. All three analyses are shown in Figure 4.

Analysis	LeT-Hold	LeT-Eliminate	LeT-None	
A_1	5	10	-	
A_2	3	8	-	
A_3	3	8	-	

Fig. 4. Expert-assigned benefits realized when LeT takes an action.

Figure 5 shows the expected costs, benefits, and *cost-benefit* ratio (CBR) of the 13 equilibria we found. From this table, we see that the equilibria that offer the best CBR are those near or below 1.0. Interestingly, each of the three expert analyses determines the same three equilibria $(E_1, E_2, \text{ and } E_5)$ with the best CBR. In each of these equilibria:

- 1) The Pakistani military ceases to provide support for **LeT**, thus avoiding the a more expensive crackdown; and
- 2) The Pakistani government does nothing to LeT.

	Expected Cost			Exp	Expected Benefit			Cost-Benefit Ratio		
Eq.	A_1	A_2	A_3	A_1	A_2	A_3	A_1	A_2	A_3	
E_1	5	4	7	10	8	8	0.5	0.5	0.88	
E_2	5	4	10	10	8	8	0.5	0.5	1.25	
E_3	13	9	15	10	8	8	1.3	1.13	1.88	
E_4	10	10	16	10	8	8	1	1.25	2	
E_5	5	4	10	10	8	8	0.5	0.5	1.25	
E_6	8	5	11	10	8	8	0.8	0.63	1.38	
E_7	18	15	21	7.5	5.5	5.5	2.4	2.73	3.82	
E_8	18	15	21	8.33	6.33	6.33	2.16	2.37	3.32	
E_9	14	12.5	18.5	5	4	4	2.8	3.13	4.63	
E_{10}	13	9.67	15.67	3.33	2.67	2.67	3.9	3.63	5.88	
E_{11}	13	9.96	15.96	3.33	2.67	2.67	3.9	3.73	5.98	
E_{12}	13	9	15	7.5	5.5	5.5	1.73	1.64	2.73	
E_{13}	10.5	7.5	13.5	7.5	5.5	5.5	1.4	1.36	2.45	

Fig. 5. Expert-assigned benefits realized when LeT takes an action.

This suggests that while the US and India should focus on (US) coordinated covert action against LeT, together with (Indian) coercive diplomacy or covert action, US diplomacy has an important role to play, namely ensuring the Pakistani military cease to provide support (no need for a crackdown) to LeT. Unfortunately, the Pakistani military has found supporting proxy armies against India a useful strategy; persuading Pakistan to abandon LeT, one of its most favored and reliable proxies, is a strategic and diplomatic challenge. Historically, Pakistan has paid a relatively low cost internationally for supporting terrorist organizations. This analysis suggests that raising those costs, both by assertively targeting LeT and also pressuring Pakistan's government, could prove effective.

IV. RELATED WORK AND CONCLUSIONS

The behavior of terror organizations, such as Lashkar-e-Taiba, is often difficult to forecast and understand due to the complex confluence of political, cultural, economic, social, and historical factors that must be considered. The complexity and dynamism of terror group behavior has made development of well-grounded statistical models difficult thus far. For instance, several works [15], [16] have developed hidden Markov models to describe how a conflict might evolve over time.

Stochastic Opponent Modeling Agents (SOMA) [17] has been used to automatically learn probabilistic behavioral model of groups like Hezbollah [6] and Hamas [7]. SOMA models were recently extracted from about 15 years of data about LeT [18]. But this does not account for the multiple stakeholders with divergent agendas associated with the group.

LeT has become the subject of a significant body of research since the Mumbai attacks in 2008. [19], [20] focus on the details of these attacks and their potential future implications. Other work [21] provides an overview of the group's recent evolution leading up to Mumbai and the future of terrorism in India and Kashmir, focusing on the role of LeT [2]. In contrast to these works which are all entirely qualitative, our work is the first treatment of LeT's behavior which explicitly quantifies the goals and motivations of these organizations and then uses the rich mathematics of game theory to understand how we might go about mitigating LeT's unfavorable actions. The idea of modeling multiple players in geopolitical conflicts using game theory is not new-Schelling [11] used it extensively in Cold War nuclear negotiations and Mesquita studied political bargaining as well using game theory [12].

We conclude by reiterating the message from our gametheoretic analysis. (i) India and the US need to join forces to exert pressure on LeT through the use of a mix of covert operations against LeT and/or coercive diplomacy with respect to Pakistan, (ii) the Pakistani military needs to be incentivized to stop providing support to LeT through an appropriate set of measures, and (iii) the Pakistani civilian government need not do anything against LeT. Not a single Nash equilibrium suggests expanding US support for Pakistan or having Pakistan's civilian government supporting LeT's social service programs.

Although models never capture everything and can often be wrong, the same is true of even the smartest of analysts. The complexity of analyzing the space of options with respect to even just the five players and relatively small set of actions we have considered is probably beyond the reach of most normal human beings. As a consequence, we strongly believe computational methods must be used in conjunction with human analysts to derive the best of both computational analytics and human subject matter expert knowledge.

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